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# Contrast Detection Probability - Implementation and use cases

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## Abstract

*The automotive industry formed the initiative IEEE-P2020 to jointly work on key performance indicators (KPIs) that can be used to predict how well a camera system suits the use cases. A very fundamental application of cameras is to detect object contrasts for object recognition or stereo vision object matching. The most important KPI the group is working on is the contrast detection probability (CDP), a metric that describes the performance of components and systems and is independent from any assumptions about the camera model or other properties. While the theory behind CDP is already well established, we present actual measurement results and the implementation for camera tests. We also show how CDP can be used to improve low light sensitivity and dynamic range measurements.*

## Introduction

The idea of Contrast Detection Probability (CDP) was first presented by Geese et.al.[1] in 2018. It was derived from the need to have a KPI that is independent from the system under test and also independent from which components are tested. So the same KPI shall be applied to describe the performance of a windshield or a lens.

As shown in the examples in Figure 1, the cause for loss of contrast can be manifold and is not only related to the camera system itself. CDP was designed to describe the performance of a camera system to reproduce contrasts, the core functionality needed for advanced algorithms in machine vision.


***Figure 1.*** *Different aspects in imaging that can lead to a contrast loss on the input side of a camera. In these examples this is fog or pollen dust on a windshield.*

Another important new aspect in automotive imaging is High Dynamic Range and the impact on system performance. As described in the IEEE-P2020 white paper [2] and shown in Figure 2, the HDR rendering process can lead to so called SNR drops. This is an effect from combining e.g. the dark part of one image with the bright part of another. The resulting SNR curve will show drops somewhere between the maximum and minimum light intensity. An example is shown in figure 3. The SNR drop can be observed in the SNR curve, a plot of the SNR vs. the light intensity. The open question is, how much impact does this have on the system performance. Even though the SNR value is a well established metric, it is very hard to derive precise system performance predictions from the SNR value. The CDP value has this possibility, as it is directly related to the system performance.

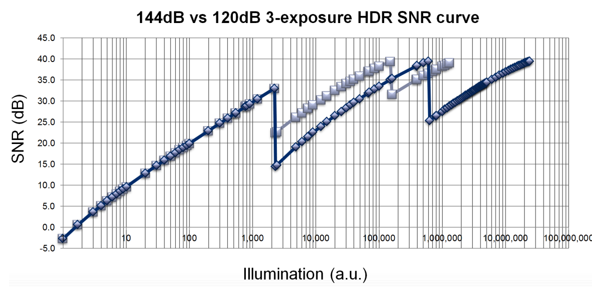


***Figure 2.*** *A typical SNR curve for HDR rendering. Depending on different sensor settings, more or less significant SNR drops occur. Source: IEEE-P2020*

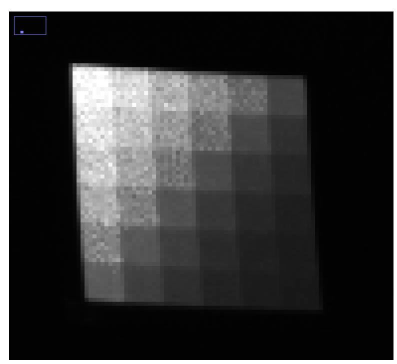

***Figure 3.*** *An example image (detail) of the effects also known as SNR drop. The noise is more dominant in the bright parts of the image compared to the dark parts. The bright part is rendered from another image than the dark part.*

## Concept of Contrast Detection Probability

A single CDP value describes the probability that two system outputs (e.g. two digital values) create a contrast from two system inputs (e.g. two luminance inputs). It is calculated from two random variables (here A and B). These random variables are the distribution of digital values for a flat-field area in an image with a known luminance. A noise free system would have a distribution where all digital values in a region of interest (ROI) equals the mean value. A real system will show, depending on noise and signal processing a distribution of values. From the two random variables A and B a new random variable K is calculated, which defines the contrast. K also has a probability density, which represents the probability to measure or detect a certain contrast. The CDP value is the area beneath the PDF, limited by a defined confidence interval.

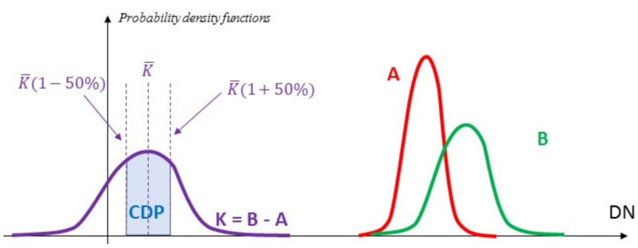


***Figure 4.*** *Calculation of CDP based on random variables A and B. The probability density function K is calculated based on the difference between A and B. CDP is calculated as the area beneath the PDF limited by a confidence interval of* $\pm 50\%$ *in this example*

The CDP value is not an absolute value, it has parameters that allow it to be tailored for the demanded use cases. So different use cases will require different calculation steps and will lead to different CDP values.

One aspect that needs to be defined is how to calculate the probability density function K. In our implementation, we choose a random pairing approach, others might be possible. Random pairing mean that random pixels from variable A and random pixels from variable B are used to calculate K. In case of a simple difference, the pixel value from a random pixel in A is subtracted from a random pixel in B. Doing this for all pixels will lead to the PDF K.

To obtain a stable PDF K, a large amount of random pairings are required. Taking multiple frames of the test scene and performing the pairing not only between two ROI in one frame, but also over multiple frames, will increase the total number of pairings and increase the stability.

Subtracting the pixel values A from B will describe the absolute difference between two patches, the use case would be to find out if the system under test can generate a difference between A and B.

Another option is to calculate a contrast value for every combination of pixels in A and B. Different options are e.g. Michelson contrast or Weber contrast. So every pair of pixels between A and B will be used to calculate a contrast value, this will generate K. The use case for this would be to check how well and within which limitations the system under test will reproduce object contrasts in an absolute sense. So if a system would increase or decrease the object contrast, it will lead to a lower CDP value. This can be required if a system is trained to classify objects based on their contrast or to find out how much impact an HDR rendering has on the object contrast.

Another aspect that needs to be derived from the use case and requirement analysis process is to define the required confidence interval to calculate CDP. Basically these are the edge cases that are still acceptable for the use case. A smaller interval will lead to lower CDP values, making it less likely that the measurement will fall into the confidence interval.

Examples for different methods to calculate K and the differences in the confidence interval are shown in figure 5.

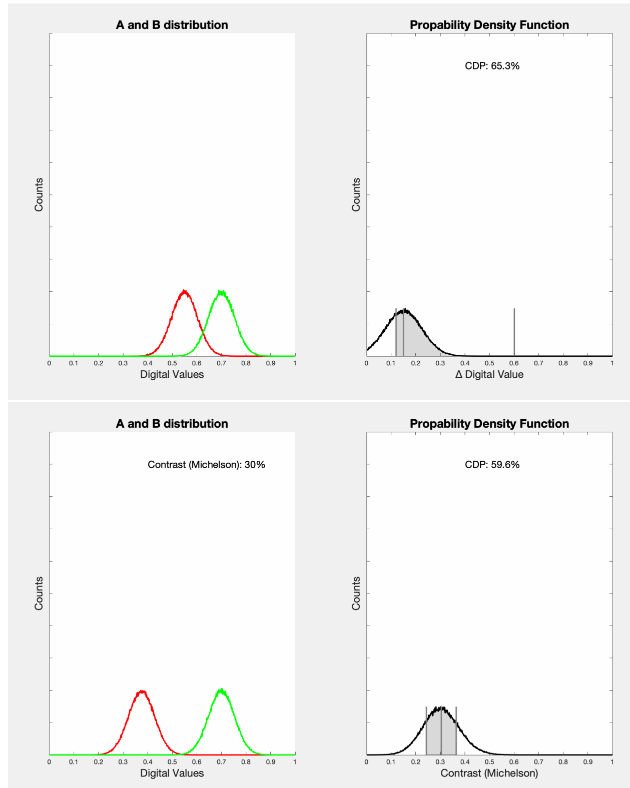


***Figure 5.*** *Two different scenarios to calculate CDP.* ***top:*** *Calculating the CDP based on simple difference. The confidence interval is not limited to the top, higher difference is always accepted* ***bottom:*** *Calculating the CDP based on Michelson contrast. The confidence interval is limiting too high and too low contrasts in the image with a confidence interval of* $\pm 50\%$

Each measurement of CDP is a function of many variables, for example: contrast and luminance. The luminance is the average of the luminance of A and B and the contrast is the contrast generated by the luminance of A and B. It is strongly adviced to measure the CDP for a large amount of combinations of contrast and luminance. As we have two variables, we can display and plot the CDP in different ways. A simple and already previously presented form is the 2D plot as shown in figure 9. Here each plot line represents a different contrast.

A new approach to present the data is to plot luminance, contrast and CDP in the same plot as 3D data, while CDP is either the height or is represented by a different color encoding. This representation was chosen to show measurement data in figure 10.

## Measurement

We choose a test setup as shown in figure 6. The device under test has to reproduce six test targets, each of them showing 36 gray patches (see Figure 7). Each of the charts is back illuminated with a dimmable flat field light source. The total contrast that can be generated is 120dB, each individual chart features a contrast of 10:1.

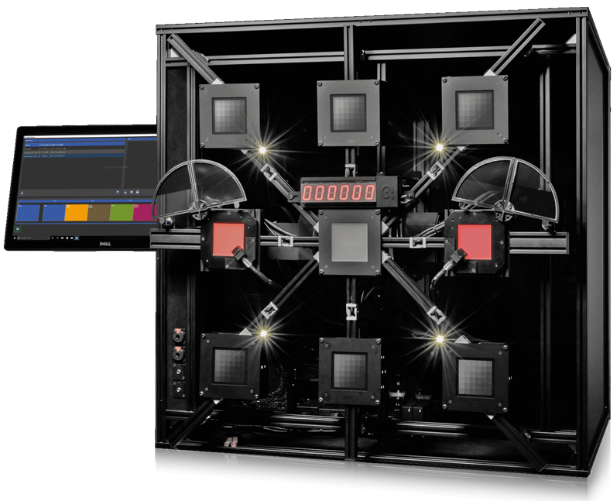

*Figure 6. The measurement device used for this test. The key components for this test are the six back illuminated test targets, three in the top row, three in the bottom row. Details of the chart shown in figure 7.*

The targets themselves generate 216 measurement points for one image. Additionally, a dynamic change of the intensity of the flat field light sources is possible as well and increases the total amount of measurement points (see Figure 8).

By gathering the image data over time we can increase the luminance of each light source by 1% for each measurement step and thus reducing the minimal available contrast steps nominally to 1%. The target with the highest luminance remains constant during this procedure to ensure an absolute reference point. There is a square correlation between the number of available luminance and the resulting measurement points. A higher number of measurement points increases the probability to find local artifacts in the measurement.

Several different devices were tested, only very few devices can be published due to promised confidentially. The results shown in figure 9 and 10 are obtained from an automotive camera with HDR mode. They show the same data in different representations.

We can clearly identify so called SNR drops, while we can now derive information how much this actually affects the potential contrast detection in the final use case. So we obtain more useful results than only an SNR curve can do.

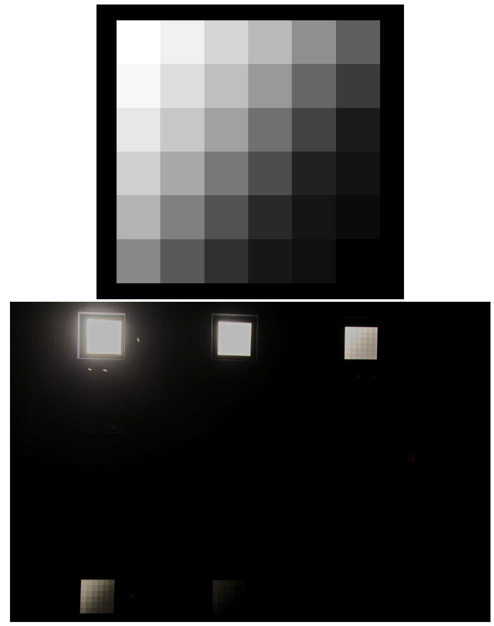

*Figure 7. Details of the used test system.* ***top:*** *One of the six identical test targets. Different intensities generated by different back illumination, one illumination per chart. The combined dynamic range of all charts is 120dB.* ***bottom:*** *An example image captured by a camera under test.*

### Linking CDP to SNR

In the annex of this paper we derive a theoretical link between CDP and SNR by utilizing several assumptions. Please note again that this calculation of a CDP value from SNR is only possible if the assumptions for the probability distributions are valid and if the system under test allows meaningful SNR measurements. Here we present the results of this theoretical derivation for Poisson-only and Gauss-only systems that are based on eq. 63 and eq. 44.

Figure 11 shows this derived connection for the noisy Poisson system and we can observe that the connection between CDP and SNR is, as expected, highly nonlinear. To reach a noteworthy single measurement detection probability (e.g. $CDP > 10\%$), an $SNR_{dB} > 10$ is needed. Of course CDP can also be applied to prefiltered signals, for example to the average of a $3 \times 3$ neighborhood to improve the detection probabilities.

Figure 12 shows the same connection between CDP and SNR considering a Gaussian approximation of the probability densities. The results are similar in the high intensity domain, however deviate in the low light domain. This shows that for Poisson dominant systems a CDP estimation based on a pure Gaussian noise assumption will lead to significant differences. Especially for low light use cases such an assumption might lead to a misjudgment of the system performance.

To finalize, a comparison of a noisy system is analyzed in Figure 13. Not depicted are $SNR_{dB} > 30$ and $CDP > 0.35$ because CDP and SNR converge to their above already discussed nonlinear connection in this range. In the low light scenarios however, SNR and CDP seem not to be compatible measures. Even though the SNR is noise compensated, $CDP$ between $10\% - 20\%$ yields to a variety of possible SNR measures e.g. $SNR_{dB} \in \{0, 5\}$, de-

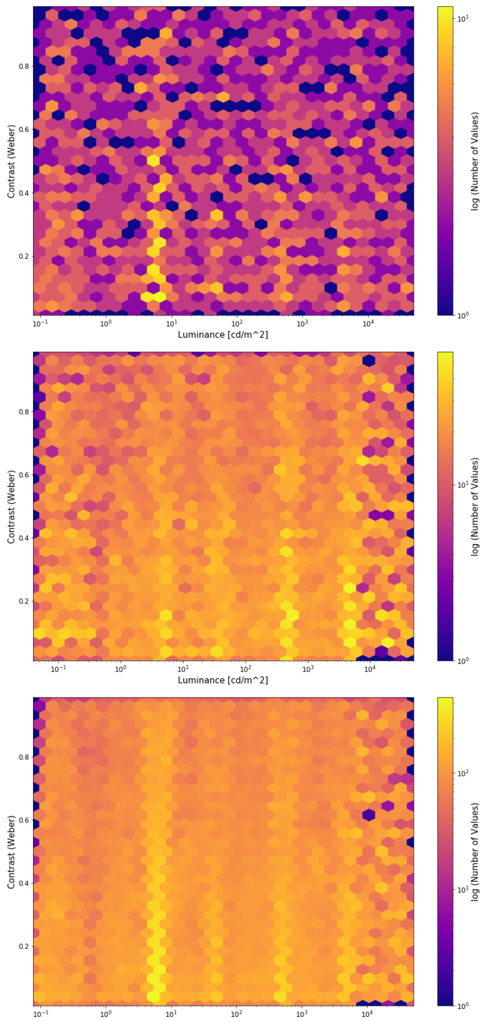


***Figure 8.*** *Per luminance and contrast available measurement points as color code for different measurement modes of the device. The amount of measurement points will increase by shifting.* ***top:*** *no shifting* ***center:*** *3 step shifting* ***bottom:*** *6 step shifting*

pending on the noise in the signal. As typical low light sensitivity tests are executed in this domain the SNR values seem not to be able to predict the CDP behavior of the system correctly. The full shape of the probability density distribution of the connected noise processes has to be considered to measure the correct CDP performance.

## Future Work

This new representation of 3D CDP visualization from fig. 10 can easily visualize how CDP measurement can also be used to derive information about dynamic range or sensitivity from a CDP measurement. Traditionally, these measurements are derived from an SNR measurement, but as the exact meaning of SNR is questionable, also the usefulness of derived metrics should be reviewed.

Considering the dynamic range of a system, such a KPI shall

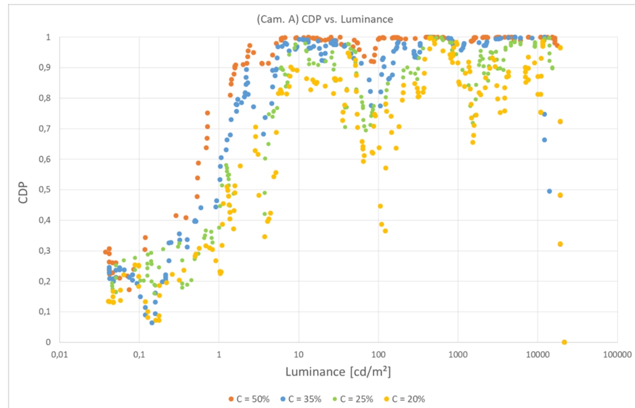


***Figure 9.*** *2D plot of CDP data from an automotive camera with HDR functionality. The different plots represent different contrast level.*

describe the range of scene luminance that the system under test can reproduce. If something is darker or brighter than the measured maximum and minimum luminance, we have a loss of information. With new HDR rendering algorithms and the already explained issues, a pure dynamic range description would exclude the so called SNR drops. We propose to use CDP to measure dynamic range related KPIs.

A dynamic range based on CDP would require to define a minimum CDP that is accepted and then apply this minimum to a CDP plot like shown in figure 10. If we set all measurement points larger than the minimum to 1 and all below the required minimum to 0, we actually have an area that defines under which conditions useful data can be obtained from the system.

A typical way to describe sensitivity of a system is the SNR10 value. This is the luminance that leads to an SNR value of 10. As we now understand, that the meaning of SNR values is limited, we should also consider to not use SNR as a metric and define other measurements.

## Conclusion

- The CDP measurement is a new KPI that is directly linked to the application and use case of the system under test.
- The relation between SNR and CDP is non-linear and depends on noise influences. Only given further assumptions, CDP be calculated directly from SNR numbers.
- CDP can be used to describe the performance of different components within the imaging chain using the same metric.
- CDP requires a well defined requirement design. The use case will define the required calculation of K, the sensitivity of detection algorithms and the robustness of trained systems will define the used confidence interval for calculating CDP.
- When using CDP for standardization, the workgroups need to provide a framework that makes the reporting of the assumptions made for calculating CDP mandatory and propose a typical parametrization for CDP.

## Acknowledgments

Thanks for the team in the iQ-Lab at Image Engineering GmbH & Co KG for performing the required tests.

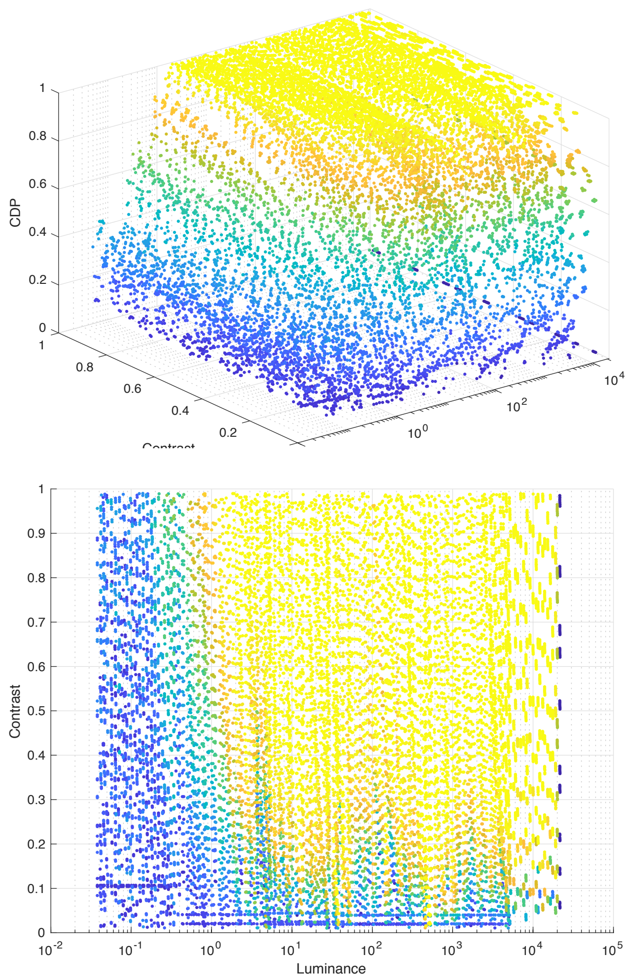


***Figure 10.*** *3D plot of CDP data from an automotive camera with HDR functionality. The X and Y axis are luminance and contrast, the Z axis is the CDP.* ***top:*** *A 3D plot* ***bottom:*** *A 2D plot with CDP represented in the color coding.*

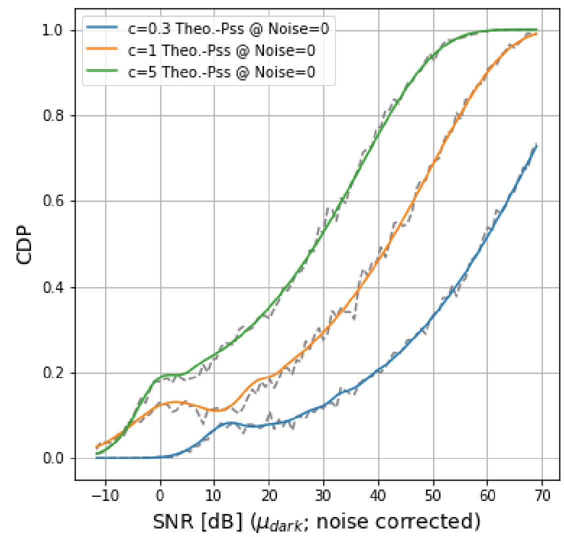


***Figure 11.*** *Intensity dependent CDP plots with $\delta_{\pm} = 0.2$ against the measured SNR values show a nonlinear connection between CDP and SNR. $SNR_{dB} > 10$ is needed to gain some significant CDP performance for the discussed use case.*

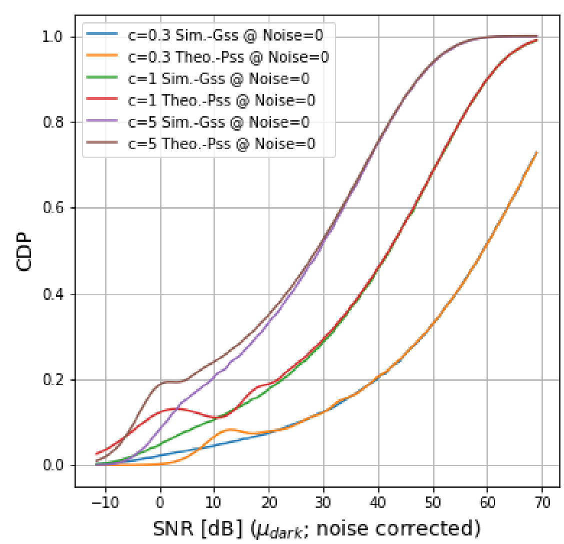


***Figure 12.*** *Intensity dependent CDP plots with $\delta_{\pm} = 0.2$ against SNR shows the nonlinear connection between CDP and SNR and the differences between the Poisson and Gaussian assumptions when calculating CDP from SNR values only. Using a Gaussian assumption to calculate CDP from SNR yields to significant deviation for $SNR_{dB} < 15$.*

## References


[1] Geese, Seger, Paollilo, "Detection Probabilities: Performance Prediction for Sensors of Autonomous Vehicles",doi:10.2352/ISSN.2470-1173.2018.17.AVM-148

[2] IEEE-SA P2020 - Automotive Image Quality Working Group, "IEEE White Paper", ISBN 9781504451130

[3] D. V. Hinkley (December 1969). "On the Ratio of Two Correlated Normal Random Variables". Biometrika. 56 (3): 635639. doi:10.2307/2334671. JSTOR 2334671.


## Author Biography

*Uwe Artmann studied Photo Technology at the University of Applied Sciences in Cologne following an apprenticeship as a photographer, and finished with the German 'Diploma Engineer'. He is now CTO at Image Engineering, an independent test lab for imaging devices and manufacturer of all kinds of test equipment for these devices. His special interest is the influence of noise reduction on image quality and MTF measurement in general.*

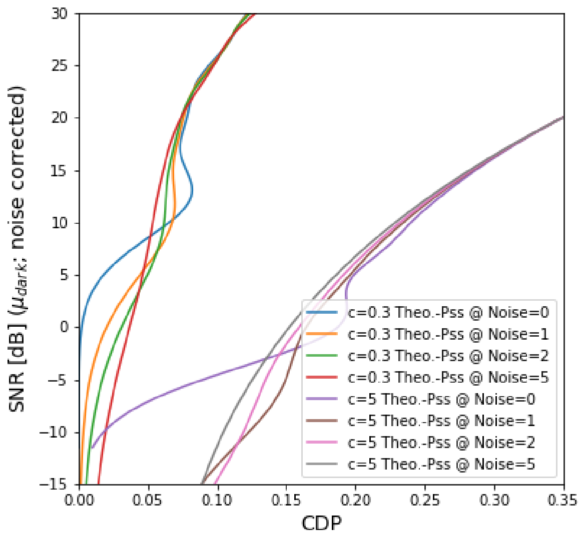


***Figure 13.*** *The CDP from SNR connection is also depending on the noise level of the signal. Only for larger detection probabilities, the CDP to SNR connection shows independent from noise. In the depicted range a calculation of the CDP performance is not possible given only the SNR measurements as input.*

*Marc Geese received a Diploma in applied physics from Frankfurt University, an MPhil in Electrical Engineering from University of Manchester and a PhD in Physics from Heidelberg University. His research focused on Complex Systems and Neural Networks applied to Image Processing tasks, Vision Chips and Computer Vision Algorithms and special computer vision algorithms for video based ADAS Systems. In his current work is as a system architect for Robert Bosch GmbH his focus is on the imaging chain consisting of optics, imager and ISP for video based ADAS systems of the next generation. His special interest is in new KPIs that serve the automotive image quality standardization and for this he is leading the subgroup no. 3 of the IEEE P2020 Working group.*

*Max Gäde is a research and development engineer at Image Engineering. After graduating from the University of Applied Sciences in Cologne with a Bachelor of Engineering degree in Media Technology, his work now focusses on test solutions for automotive imaging.*

# Annex A

The link between SNR and CDP requires a longer list of assumptions and calculation steps. As these would make the reading of the paper more complex, we show them in this annex.

## *Assumptions to link CDP to component level SNR*

All measurement processes and the photon generation itself are based on random processes which introduce noise into the measurements.

We consider a system under test (SUT), for example a complete imaging chain or any other applicable subsystem. Figure 14 shows such a system with two input stimuli $R$, the system function $s$ and the output measurement $m$.

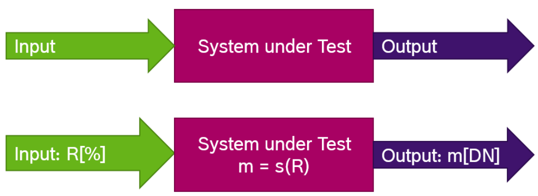


*Figure 14. System under test with reflectance input $R$, and digital output $m$. The illuminance conditions are here considered as part of the system.*

As described in the CDP definition, two inputs are needed to measure CDP, which can be generated either by a high and low light emittance, or by a high and low reflectance that is illuminated by a constant light source. Assuming these input signals as ideal we would observe delta peaks in their probability density as shown in figure 15.

Measuring the two input reflectances, we obtain a set of bright and dark measurements (e.g. $\{m_{\max,i}\}$, $\{m_{\min,i}\}$). Of course these measurements depend, among other variables, strongly on the illumination that has to be considered part of the SUT.

The measurements are not deterministic due to their noise dependencies (e.g. photon flux Poisson noise, dark currents in the image sensors etc.). Consequently, we obtain two probability density distributions which belong to the random variables $M_{\text{bright}}$ and $M_{\text{dark}}$, as shown in figure 15.

Summarized figure 15 shows the measurement process and the obtained probability distributions.

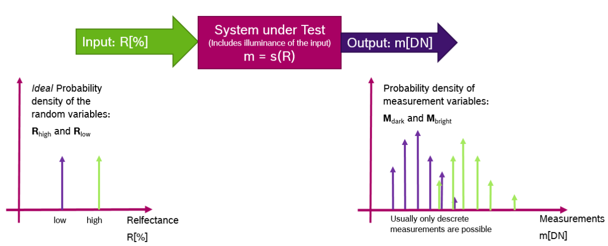


*Figure 15. Probability density distributions of the input and output of a system. The system measures two reflectance differences and contains a lightsource to illuminate the reflectances.*

**There are typical system assumptions that simplify realizations of experimental setups and theoretical considerations:**

1. The system consists of (at least locally) spatial identical, independent subsystems.
2. The system consists of (at least for a small time period) temporal identical, independent subsystems.

With the first assumption, image sensors can capture multiple measurements from a ROI that is small enough to fulfill the assumption. The second assumption can be used to capture the measurements from different (temporal) images that contribute to the above measurements $m$.

The outputs are usually in different units compared to the input. A typical imaging system has outputs in digital numbers $DN$. The conversion between the input reflectances in % (or alternatively the reemitted illuminance in $\frac{cd}{m^2}$) and the output in $DN$ is given by the system function $s$.

**The system function $s$ is also approximated with the following assumption:**

- If the measured contrast is chosen small enough, the system function is considered to be a continuous function that can be interpolated from the obtained input and output data.

For systems with local adaptive tone mapping or systems with active exposure control, the above assumption is also true, but only for a small spatial neighborhood or a (very small) time period. A generalized system function should consider at least the following variables, however, more variables could be included as well (e.g. Temperature):

$$\text{system function } s: \quad m[DN] = s(R[\%], t, (x,y)) \tag{1}$$

Figure 16 shows the estimation procedure for a given **constant** area. Note: If the system function changes locally due to local tonemapping or temporally due to exposure time shifts, the procedure needs to be repeated for each operation point of interest.

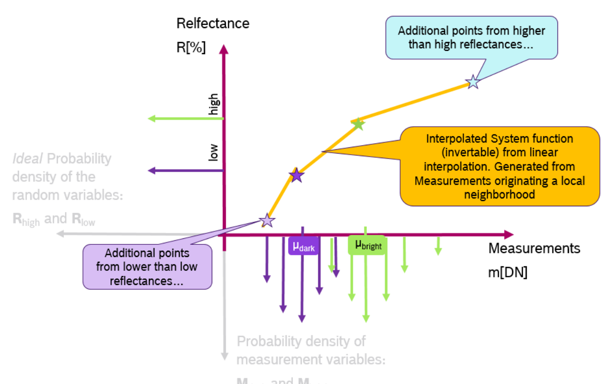


*Figure 16. Estimating the system function for a given **constant** domain of the system (e.g. fixed tonemapping in the area of the measurements and fixed exposure time for all of the measurements)*

**Contrast measurements:** From the obtained measurements the measured contrasts $c_{\text{meas.}}$ can be calculated the following way:

1. Transform the measurements from the measured units (e.g. $DN$) back into the input domain by using the inverse system function $s^{-1}$ Figure 17 shows how the measurements' probability distribution can be distorted when transferred back into the input domain.
2. Picking randomly a bright and a dark measurement.

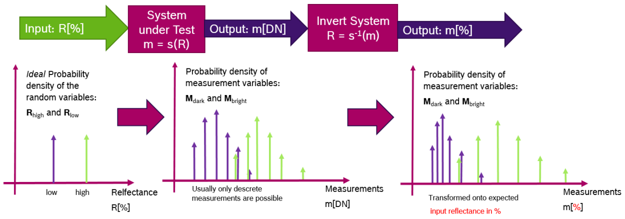


***Figure 17.*** *Probability density distributions of the possible measurements after transferring back to the input domain*

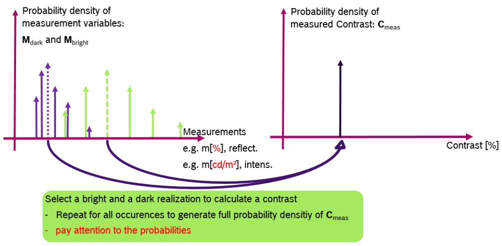


***Figure 18.*** *Picking up a random bright and a random dark measurement to calculate the measured contrast between these two.*

3. Calculate the measured contrast $c_{\text{meas.}}$ with the Weber contrast equation (See figure 18).

Executing the above steps with many different measurement values, the calculated contrasts $c_{\text{meas.}}$ form a distribution of the new random variable $\mathrm{C}_{\text{meas.}}$. See figure 19 for an illustration.

We have to accept that the measured contrast has small derivations $\varepsilon_+$ and $\varepsilon_-$ from the assumed input contrast $c_{\text{in}}$. Depending on the application either large or almost infinite small deviations can be chosen to be acceptable.

**Mathematical definition of CDP:** Mathematically expressed we define the contrast detection probability:

$$\mathrm{C}_{c_{\text{in}}}\mathrm{DP} = \mathrm{Prob}\left((c_{\text{in}} - \varepsilon_-) < c_{\text{meas.}} < (c_{\text{in}} + \varepsilon_+)\right) \tag{2}$$

Figure 20 shows an illustration on how the CDP value from eq. 2 can be extracted from the measured contrast values.

Concluding, figure 21 summarizes the workflow for CDP measurements as a block diagram.

## *Intensity dependent CDP*

Given a defined reflectance contrast $c_{\text{in}}$, CDP has a dependence on many other variables:

**Electrical System Configuration:** Which is usually **easy** to keep constant, e.g. by not changing exposure time and other registers.

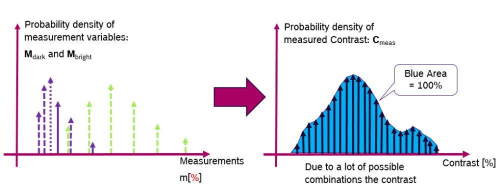


***Figure 19.*** *Many randomly selected contrast measurements form a probability distribution, which belongs to the contrast's random variable ($C_{meas.}$).*

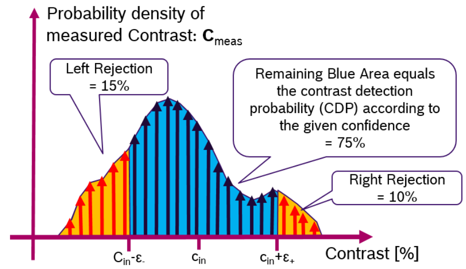


***Figure 20.*** *Probability density distributions of the input and output of a system. The system measures two reflectance differences and contains a light source to physically illuminate the reflectances.*

**Temperature:** Is controllable with **medium** difficulty, however more than one *temperature constant* operation condition is typical.

**Luminance:** Is **hard/impossible** to control, as with a changing scene, the system in input varies without control.

Given this motivation, the luminance dependency for given operating temperatures comes at first priority for investigations.

We consider an ideal image sensor, which produces for each detected photon one electron and then counts the electrons one by one. The photon flux obeys a Poisson statistic and for a given contrast $c_{\text{in}}$ the bright and the dark patch of the contrast behave as Poisson noise generators:

$$\mu_{\text{bright}} = (1 + c_{\text{in}})\mu_{\text{dark}} \tag{3}$$

$$\mathscr{P}_{\mu_{\text{dark}}}(n) = \frac{{\mu_{\text{dark}}}^n}{n!} e^{-\mu_{\text{dark}}} \tag{4}$$

$$\mathscr{P}_{\mu_{\text{bright}}}(n) = \frac{{\mu_{\text{bright}}}^n}{n!} e^{-\mu_{\text{bright}}} \tag{5}$$

with $n$ as the number of photons in the given process.

In our ideal system we consider:

$$m[DN] = s(I, R) = 1\frac{DN}{e} \cdot \underbrace{1\frac{e}{ph} \cdot I[ph.] \cdot R[\%]}_{\text{generated Poissonian electrons}} \tag{6}$$

Starting with the definition of CDP, we can derive a theoretical forecast for an ideal Poisson-only system:

$$\begin{aligned}\mathrm{C}_c\mathrm{DP} &= \Pr\left((c_{\text{in}} - \varepsilon_-) < \mathbf{c}_{\text{meas.}} < (c_{\text{in}} + \varepsilon_+)\right) && (7)\\ &= \Pr\left(\mathbf{c}_{\text{meas.}} < (c_{\text{in}} + \varepsilon_+)\right) \ldots \\ &\ldots - \Pr\left(\mathbf{c}_{\text{meas.}} < (c_{\text{in}} - \varepsilon_-)\right) && (8)\end{aligned}$$

With $\varepsilon_+, \varepsilon_-, c_{\text{in}}$ as given, we can express $c_{\text{meas.}}$ depending on the random variable based system model.

$$\begin{aligned}\mathbf{c}_{\text{meas.}} &= \frac{\mathbf{m}_{\text{bright}}}{\mathbf{m}_{\text{dark}}} - 1 && (9)\\ &= \frac{\mathbf{e}_{\text{bright}}}{\mathbf{e}_{\text{dark}}} - 1 \quad \text{based on eq. 6} && (10)\end{aligned}$$

For the sake of notation: $\mathbf{e}_{\text{bright}}$ and $\mathbf{e}_{\text{dark}}$ and $\mathbf{c}_{\text{meas.}}$ are **Poisson random variables** and $e_{\text{bright}}$ and $e_{\text{dark}}$ and $c_{\text{meas.}}$ are their respective **realizations**.

We can continue to derive CDP by:

$$\begin{aligned}&\Pr\left(\mathbf{c}_{\text{meas.}} < (c_{\text{in}} + \varepsilon_+)\right) \quad \ldots \\ &= \Pr\left(\frac{\mathbf{e}_{\text{bright}}}{\mathbf{e}_{\text{dark}}} - 1 < (c_{\text{in}} + \varepsilon_+)\right) && (11)\\ &= \Pr\left(\mathbf{e}_{\text{bright}} < (c_{\text{in}} + \varepsilon_+ + 1)\mathbf{e}_{\text{dark}}\right) && (12)\end{aligned}$$

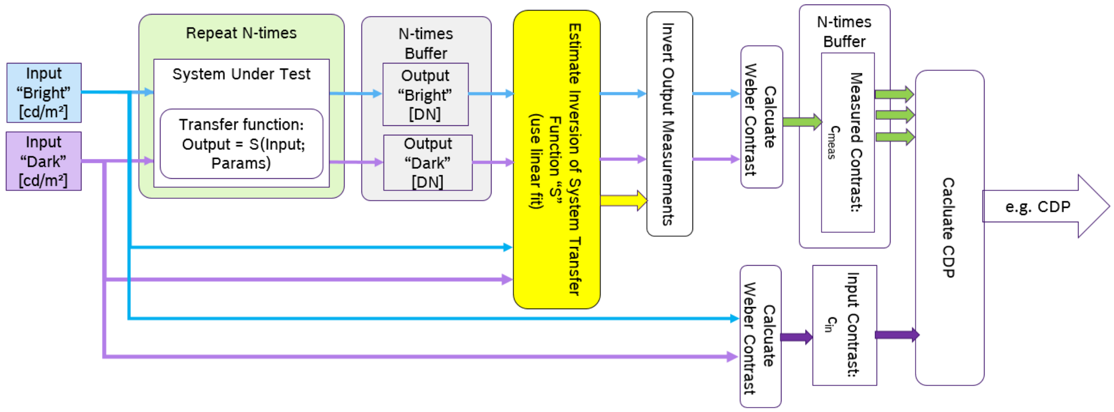


***Figure 21.*** *The summarized workflow for CDP measurement for any given system under test which results in one measurement value of CDP.*

The above equation can only be fulfilled if $\mathbf{e}_{\text{dark}} > 0$, which is due to the quantized nature of electrons and photons

$$\mathbf{e}_{\text{dark}} >= 1 \tag{13}$$

To evaluate the full probability the summation over the bright intensity need to be limited depending on the current dark patches electron count (d = dark, b = bright):

$$\ldots = \sum_{e_{\text{d}}=1}^{\infty} \sum_{e_{\text{b}}=0}^{\lfloor (c_{\text{in}}+\varepsilon_{+}+1)e_{\text{d}} \rfloor} \frac{\mu_{\text{b}}{}^{e_{\text{b}}}}{e_{\text{b}}!} e^{-\mu_{\text{b}}} \cdot \frac{\mu_{\text{d}}{}^{e_{\text{d}}}}{e_{\text{d}}!} e^{-\mu_{\text{d}}} \tag{14}$$

$$= e^{-\mu_{\text{d}}} e^{-\mu_{\text{b}}} \sum_{e_{\text{d}}=1}^{\infty} \sum_{e_{\text{b}}=0}^{\lfloor (c_{\text{in}}+\varepsilon_{+}+1)e_{\text{d}} \rfloor} \frac{\mu_{\text{b}}{}^{e_{\text{b}}}}{e_{\text{b}}!} \frac{\mu_{\text{d}}{}^{e_{\text{d}}}}{e_{\text{d}}!} \tag{15}$$

This goes in the same methodology for the second term:

$$\Pr\left(\mathbf{c}_{\text{meas.}} < (c_{\text{in}} - \varepsilon_{-})\right) \quad \ldots$$

$$= \Pr\left(\frac{\mathbf{e}_{\text{b}}}{\mathbf{e}_{\text{d}}} - 1 < (c_{\text{in}} - \varepsilon_{-})\right) \tag{16}$$

$$= e^{-\mu_{\text{d}}} e^{-\mu_{\text{b}}} \sum_{e_{\text{d}}=1}^{\infty} \sum_{e_{\text{b}}=0}^{\lfloor (c_{\text{in}}-\varepsilon_{-}+1)e_{\text{d}} \rfloor} \frac{\mu_{\text{b}}{}^{e_{\text{b}}}}{e_{\text{b}}!} \frac{\mu_{\text{d}}{}^{e_{\text{d}}}}{e_{\text{d}}!} \tag{17}$$

Combined the CDP from eq. 8 gets into the form for Poisson-only noise:

$$\mathrm{C}_{c_{\text{in}}}\mathrm{DP} = \Pr\left(\mathbf{c}_{\text{m}} < (c_{\text{in}} + \varepsilon_{+})\right) - \Pr\left(\mathbf{c}_{\text{m}} < (c_{\text{in}} - \varepsilon_{-})\right) \tag{18}$$

$$= e^{-\mu_{\text{d}}} e^{-\mu_{\text{b}}} \sum_{e_{\text{d}}=1}^{\infty} \sum_{e_{\text{b}}=0}^{\lfloor (c_{\text{in}}+\varepsilon_{+}+1)e_{\text{d}} \rfloor} \frac{\mu_{\text{b}}{}^{e_{\text{b}}}}{e_{\text{b}}!} \frac{\mu_{\text{d}}{}^{e_{\text{d}}}}{e_{\text{d}}!} \ldots$$

$$\ldots - e^{-\mu_{\text{d}}} e^{-\mu_{\text{b}}} \sum_{e_{\text{d}}=1}^{\infty} \sum_{e_{\text{b}}=0}^{\lfloor (c_{\text{in}}-\varepsilon_{-}+1)e_{\text{d}} \rfloor} \frac{\mu_{\text{b}}{}^{e_{\text{b}}}}{e_{\text{b}}!} \frac{\mu_{\text{d}}{}^{e_{\text{d}}}}{e_{\text{d}}!} \tag{19}$$

$$= e^{-\mu_{\text{d}}} e^{-\mu_{\text{b}}} \sum_{e_{\text{d}}=1}^{\infty} \frac{\mu_{\text{d}}{}^{e_{\text{d}}}}{e_{\text{d}}!} \sum_{e_{\text{b}}=\lceil (c_{\text{in}}-\varepsilon_{-}+1)e_{\text{d}} \rceil}^{\lfloor (c_{\text{in}}+\varepsilon_{+}+1)e_{\text{d}} \rfloor} \frac{\mu_{\text{b}}{}^{e_{\text{b}}}}{e_{\text{b}}!} \tag{20}$$

To be able to calculate this equation for larger electron or photon counts we need to use the Stirling approximation:

$$n! \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^{n} \tag{21}$$

$$\log n! \sim \frac{1}{2} \log 2\pi n + \log n^{n} - \log e^{n} \tag{22}$$

$$\sim n \log n - n + \frac{1}{2} \log 2\pi n \tag{23}$$

We include all exponential functions into their log approximation to get the best numerical stability (including $e^{-\mu_{\text{d}}} e^{-\mu_{\text{b}}}$), and obtain:

$$f(e, \mu) = \exp\left(\log\left(e^{-\mu} \cdot \frac{\mu^{e}}{e!}\right)\right) \tag{24}$$

$$= \exp\left(e \log \mu - e \log e + e - \frac{1}{2} \log 2\pi e - \mu\right) \tag{25}$$

$$CDP = \sum_{e_{\text{d}}=1}^{\infty} f(e_{\text{d}}, \mu_{d}) \sum_{e_{\text{b}}=\lceil (c_{\text{in}}-\varepsilon_{-}+1)e_{\text{d}} \rceil}^{\lfloor (c_{\text{in}}+\varepsilon_{+}+1)e_{\text{d}} \rfloor} f(e_{\text{b}}, \mu_{b}) \tag{26}$$

For numerical evaluation we do not need to sum over all $e_b$ and $e_d$, but limit the summation to a certain number of standard deviations beyond of which we consider the probability contribution as 0”. The range between 0 and 10 is however not excluded from summation as it does not yield to computational efforts:

$$e_{\text{d,start}} = \max\{0, \mu_{\text{d}} - N_{d,\text{start}} \cdot \sqrt{\mu_{\text{d}}}\} \tag{27}$$

$$e_{\text{d,stop}} = \max\{10, \mu_{\text{d}} + N_{d,\text{stop}} \cdot \sqrt{\mu_{\text{d}}}\} \tag{28}$$

$$e_{\text{b,start}} = \max\{0, \mu_{\text{d}} - N_{b,\text{start}} \cdot \sqrt{\mu_{\text{d}}}\} \tag{29}$$

$$e_{\text{b,stop}} = \max\{10, \mu_{\text{d}} + N_{b,\text{stop}} \cdot \sqrt{\mu_{\text{d}}}\} \tag{30}$$

for the bright variables we also have to update the limits given by the above deliberations that yield to the correct summing from eq. 20

$$e_{\text{b,start}} = \max\{e_{\text{b,start}}, \lceil (c_{\text{in}} - \varepsilon_{-} + 1)e_{\text{d}} \rceil\} \tag{31}$$

$$e_{\text{b,stop}} = \min\{e_{\text{b,stop}}, \lfloor (c_{\text{in}} + \varepsilon_{+} + 1)e_{\text{d}} \rfloor\} \tag{32}$$

Here only $\mu_{\text{b}}$ is an unknown variable and can be eliminated by using the contrast definition:

$$\mu_{\text{b}} = (c_{\text{in}} + 1)\mu_{\text{d}} \tag{33}$$

Further, the limits $\varepsilon_{-}$ and $\varepsilon_{+}$ are currently given as absolute limits w.r.t. to the given contrast. While this a good choice to evaluate special contrast detection tasks, these limits can also be expressed as relative percentages to the incoming contrast, which allows a more generic evaluation:

$$c_{\text{in}} + \varepsilon_{+} = c_{\text{in}} + \delta_{+} c_{\text{in}} \quad \Rightarrow \quad \varepsilon_{+} = \delta_{+} \cdot c_{\text{in}} \tag{34}$$

And in the same way for $\varepsilon_-$:

$$c_{\text{in}} - \varepsilon_- = c_{\text{in}} - \delta_- c_{\text{in}} \quad \Rightarrow \quad \varepsilon_- = \delta_- \cdot c_{\text{in}} \tag{35}$$

Which gives in the above formula in the following way:

$$CDP = \sum_{e_{\text{d}}=1}^{\infty} f(e_{\text{d}}, \mu_d) \sum_{e_{\text{b}}=\lceil (c_{\text{in}}(1-\delta_-)+1)e_{\text{d}} \rceil}^{\lfloor (c_{\text{in}}(1+\delta_+)+1)e_{\text{d}} \rfloor} f(e_{\text{b}}, \mu_b) \tag{36}$$

Figure 22 shows some plots of the derived equation against simulations of this approach. It can be well observed that the simulation fits the theoretical deliberations. We can observe that the ideal Poisson-only noise system has a CDP that in general behaves as expected:

- Higher contrast leads to higher CDP
- More photons or electrons, leads to higher CDP

Further, CDP has the ability to detect the systems signal quantization, especially if large contrasts are chosen. This is depicted in the green circle and shows best for the $c = 7500\%$ Weber contrast. This yields to a non-monotonic function and depending on the system under test this effect can be based by each quantization process of the system: The photonic quantization, the photo-electrons or the ADCs. This property can be used to analyze the systems behavior in detail.

As a side remark: For special experimental setups, e.g monochromatic light and well defined contrasts, this behavior could be used to measure basic physical constants.

We propose the following explanation of this behavior: CDP has obviously a high value if the photon counts between bright and low signal fits best to the input contrast. This behavior is attached to single photon steps for intensities where the dark patch reaches an intensity of single digit photon counts (or other quantization mechanisms). Utilization of this behavior will be part of the further work.

For very low light intensities, CDP of the larger contrast is below the CDP of the lower contrast. This a non-intuitive results but is explained by the fact that at low light level a photon count of either one or zero photons is present. As larger contrasts demand larger photon counts to fit into the CDP confidence interval, compared to low contrasts the behavior makes sense. To cross check this explanation, figure 23 shows that this behavior goes indeed away if the lower acceptance limit is set to $\delta_- = 90\%$ for example.

**Non Summative Approximation** Given the equations above and the fact that Poissonian processes can be approximated by Gaussian Processes if their expectation value is high enough it is also possible to get a non-summative approximation by using a ratio of Gaussian processes.

$$CDP_{\text{Gauss}} \quad \text{approximate by:} \quad c_{\text{in,Gauss}} \approx \frac{\mathbf{e}_{\text{b,Gauss}}}{\mathbf{e}_{\text{d,Gauss}}} - 1 \tag{37}$$

David Hinkley's work can be used to derive the equations [3]. For this work, only a simulation was considered which shows in figure 24 a significant derivation from the Poissonian approach. This hints that (as always) Gaussian approximations shall only be used if their possible effects errors have been evaluated in detail.

## CDP approximation by SNR and EMVA1288

Having the above equations. We can start to include more complex system models. For example the EMVA1288 standard gives us a model to include some of the basic effects.

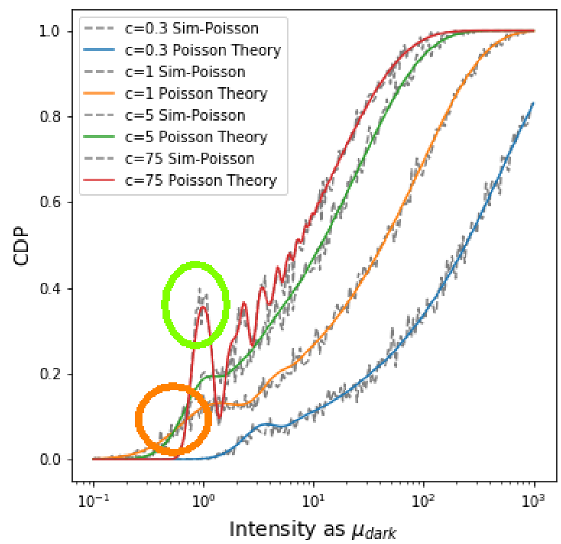


***Figure 22.*** *Intensity dependent CDP ($\delta_\pm = 0.25$) of different contrasts compared to typical simualtion results (dashed lines). Green Circle: For large contrasts, the quantization of the underlying signal which is based on photons and electrons becomes visible. Orange Circle: Due to a lack of signal quants, large contrasts cannot be realized below a certain threshold. See text for discussion.*

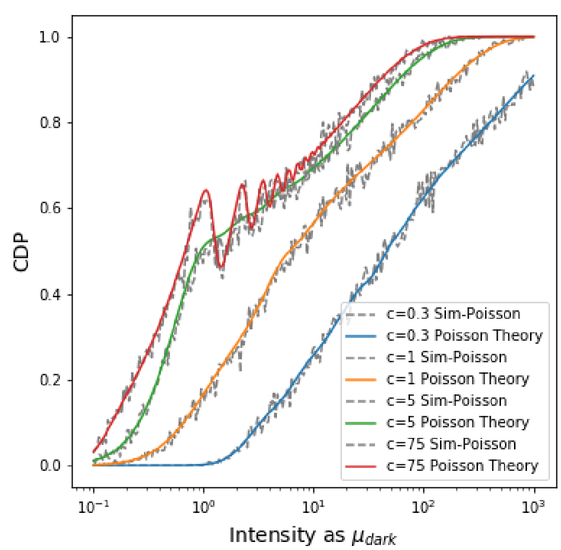


***Figure 23.*** *Intensity dependent CDP ($\delta_- = 0.9, \delta_+ = 0.25$) shows that CDP curve crossing at low light does not happen in this case as described in the discussion of this effect.*

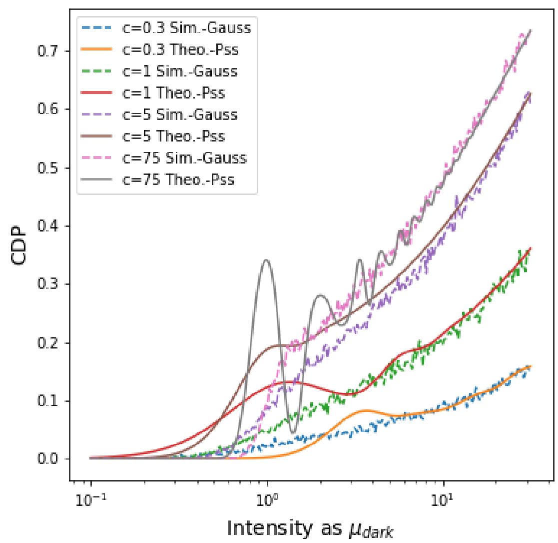


*Figure 24.* *Intensity dependent CDP ($\delta_- = 0.5, \delta_+ = 0.5$) by simulation with a Gaussian approximation. The differences become clearly visible in the low light domain around* 1 *to* 10 *photons.*

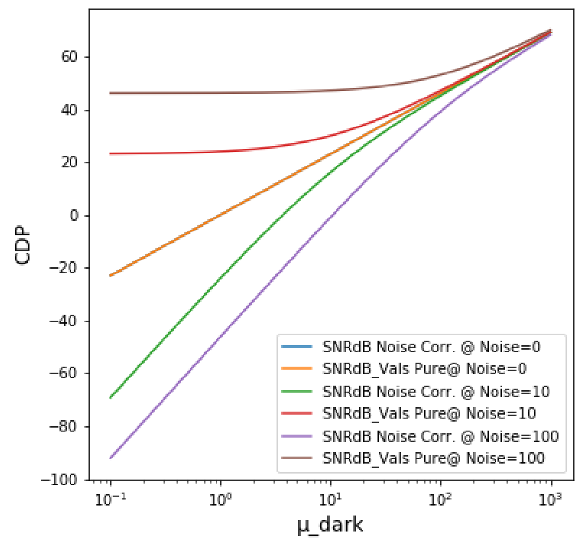


*Figure 25.* *Intensity dependent SNR calculations for pure Poisson processes. If the known noise sources are not considered, the SNR values give large despite a noise degraded signal*

We can use the Poissonian properties to set the above result into a dependency to SNR:

$$\sigma^2 = \mu \quad \text{as variance} \tag{38}$$

$$\sigma = \sqrt{\mu} \quad \text{as standard deviation} \tag{39}$$

$$\text{SNR} = \frac{\mu}{\sigma} = \sqrt{\mu} \tag{40}$$

$$\text{SNR}_{\text{dB}} = 20 \cdot \log\left(\frac{\mu}{\sigma}\right) = 10 \cdot \log(\mu) \tag{41}$$

Here we can choose either the bright or the dark number of electrons to calculate the SNR. As the contrast detection usually demands for a detection of the dark patch, I propose to use the dark patches expectation value to calculate the SNR. The results are already shown in the above plots.

If we want to calculate SNR in the domain of digital numbers *DN* we need to include the possible more complex system function *S* and include this into the above considerations.

## Including Veiling Glare and Dark Current

If we want to include veiling glare and dark current we follow the assumption from our Electronic Imaging AVM paper [1] that all further contributions are additive Poisson noise sources. For the dark current and the veiling glare this is obvious. For other electron generating sources our last paper showed how they could be handled as Poisson sources as well.

As Poisson processes are additive we obtain for expectation value and standard deviation the following:

$$\text{Signal + Noise:} \quad \sigma^2_{\text{total}} = \sigma^2_{\text{Signal}} + \sigma^2_{\text{Noise}} \tag{42}$$

$$\text{Poissonian Variance:} \quad \sigma^2_{\text{total}} = \mu_{\text{total}} = \mu_{\text{Signal}} + \mu_{\text{Noise}} \tag{43}$$

However, if we want to calculate SNR, we have to try to compensate for the known noise sources otherwise the SNR gives large numbers even in the absence of any signal. See figure 25 for an illustration. The following equations show how to calculate the noise corrected SNR for our given model:

$$\text{SNR} = \frac{\mu_{\text{Signal}}}{\sigma_{\text{total}}} = \frac{\mu_{\text{total}} - \mu_{\text{Noise}}}{\sqrt{\mu_{\text{Signal}} + \mu_{\text{Noise}}}} \tag{44}$$

$$\text{SNR}_{\text{dB}} = 20 \cdot \log\left(\frac{\mu_{\text{total}} - \mu_{\text{Noise}}}{\sqrt{\mu_{\text{Signal}} + \mu_{\text{Noise}}}}\right) \tag{45}$$

## A Poisson-only Systems' CDP with given Noise

Extending our above approach for Poission systems to be polluted with an additive noise, we also have to correct the noise mean value to obtain a good CDP derivation. In full extend this should be done by inverting the system function *S*. To showcase the implication onto an ideal noise deteriorated system, we can execute this task with the following steps:

$$\mathbf{e}_{\text{b}} \quad : \text{bright signal including noise} \tag{46}$$

$$\mathbf{e}_{\text{d}} \quad : \text{dark signal including noise} \tag{47}$$

$$\mu_{\text{N}} \quad : \text{known expectation value of additive noise} \tag{48}$$

This allows us to rewrite to eq. 18 in the following way:

$$\text{Prob}\left(\mathbf{c}_{\text{meas.}} < (c_{\text{in}} + \varepsilon_+)\right) \quad \dots$$

$$\dots \quad = P\left(\frac{\mathbf{e}_{\text{b}} - \mu_{\text{N}}}{\mathbf{e}_{\text{d}} - \mu_{\text{N}}} - 1 < (c_{\text{in}} + \varepsilon_+)\right) \tag{49}$$

$$= P\left(\mathbf{e}_{\text{b}} - \mu_{\text{N}} < (c_{\text{in}} + \varepsilon_+ + 1)(\mathbf{e}_{\text{dark}} - \mu_{\text{N}})\right) \tag{50}$$

$$= P\left(\mathbf{e}_{\text{b}} < ((c_{\text{in}} + \varepsilon_+ + 1)(\mathbf{e}_{\text{dark}} - \mu_{\text{N}}) + \mu_{\text{N}}\right) \tag{51}$$

$$= P\left(\mathbf{e}_{\text{b}} < (c_{\text{in}} + \varepsilon_+ + 1)\mathbf{e}_{\text{dark}} - \mu_{\text{N}}(c_{\text{in}} + \varepsilon_+)\right) \tag{52}$$

and similar for the second term:

$$\begin{aligned}
&\mathrm{Prob}\left(\mathbf{c}_{\text{meas.}} < (c_{\text{in}} - \varepsilon_-)\right) \quad \dots \\
&\quad = \mathrm{Prob}\left(\frac{\mathbf{e}_\text{b} - \mu_\text{N}}{\mathbf{e}_\text{d} - \mu_\text{N}} - 1 < (c_{\text{in}} - \varepsilon_-)\right) && (53)\\
&\quad = \mathrm{Prob}\left(\mathbf{e}_\text{b} < (c_{\text{in}} - \varepsilon_- + 1)\mathbf{e}_\text{d} - \mu_\text{N}(c_{\text{in}} - \varepsilon_-)\right) && (54)
\end{aligned}$$

In analogy to the previous derivation, the Poisson limitation to values between 0 and $\infty$ has again an influence on the starting value of $\mathbf{e}_\text{d}$ in the summation:

$$\begin{aligned}
&\mathbf{e}_\text{b} \geq 0 && (55)\\
&\Rightarrow 0 < (c_{\text{in}} - \varepsilon_- + 1)\mathbf{e}_\text{d} - \mu_\text{N}(c_{\text{in}} - \varepsilon_-) && (56)\\
&\mathbf{e}_\text{d} > \frac{\mu_\text{N}(c_{\text{in}} - \varepsilon_-)}{(c_{\text{in}} - \varepsilon_- + 1)} && (57)
\end{aligned}$$

Thus we have starting values for $\mathbf{e}_\text{d}$ for the high and low probability borders:

$$\begin{aligned}
D_{\text{start,low}} &= \left\lceil \frac{\mu_\text{N}(c_{\text{in}} - \varepsilon_-)}{(c_{\text{in}} - \varepsilon_- + 1)} \right\rceil && (58)\\
D_{\text{start,high}} &= \left\lceil \frac{\mu_\text{N}(c_{\text{in}} + \varepsilon_+)}{(c_{\text{in}} + \varepsilon_+ + 1)} \right\rceil && (59)
\end{aligned}$$

Combined we obtain for the CDP evaluation from eq. 8 the same form but with adapted borders:

$$\begin{aligned}
B_{\text{stop,low}} &= \lceil (c_{\text{in}} - \varepsilon_- + 1)e_\text{d} - \mu_\text{N}(c_{\text{in}} - \varepsilon_-) \rceil && (60)\\
B_{\text{stop,high}} &= \lceil (c_{\text{in}} + \varepsilon_+ + 1)e_\text{d} - \mu_\text{N}(c_{\text{in}} + \varepsilon_+) \lceil && (61)\\
\mathrm{C}_{c_{\text{in}}}\mathrm{DP} &= \mathrm{Prob}\left(\mathbf{c}_{\text{meas.,noise}} < (c_{\text{in}} + \varepsilon_+)\right) \dots \\
&\dots \quad - \mathrm{Prob}\left(\mathbf{c}_{\text{meas.,noise}} < (c_{\text{in}} - \varepsilon_-)\right) && (62)\\
&= \sum_{e_\text{d}=D_{\text{start,high}}}^{\infty} f(e_\text{d}, \mu_d) \sum_{e_\text{b}=0}^{B_{\text{stop}}} f(e_\text{b}, \mu_b) \dots \\
&\dots \quad - \sum_{e_\text{d}=D_{\text{start,low}}}^{\infty} f(e_\text{d}, \mu_d) \sum_{e_\text{b}=0}^{B_{\text{stop,high}}} f(e_\text{b}, \mu_b) && (63)
\end{aligned}$$

Figure 26 shows how CDP degrades if noise is added to the signal. There are many different parametrization options for CDP. While some implications have been discussed in the main text of this article, we leave a further discussion to future work.

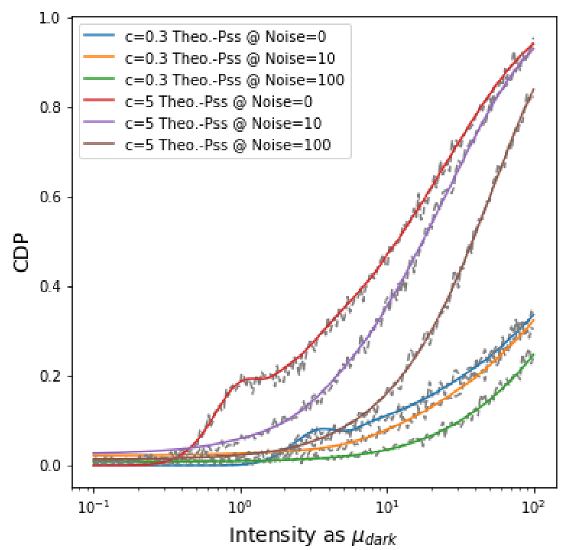


***Figure 26.*** *Intensity dependent CDP plots of $c = 0.3$ and $c = 5$ with added noise in the strength of $0, 10$ or $100$ electrons.*